Unlocking Determinants

Class 12 NCERT -Exercise 4.1

Your shortcut to solving all 8 questions — explained simply.

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Exercise 4.1

This exercise focuses on evaluating determinants.

Question Breakdown:

- Q1 to Q3: 2×2 matrices — Evaluate the Determinant
- Q4 to Q6: 3×3 matrices — Evaluate the Determinant
- Q7 & Q8: 2×2 matrices — Find the value of x

What is a Determinant

- The determinant is a **number** calculated from a **square matrix** (like 2×2, 3×3, etc.).
- It's denoted as $|A|$ or $\det(A)$ for a matrix A.

How to Evaluate it.

- For a 2×2 Matrix, use a simple formula:- $a_1 b_2 - a_2 b_1$
- For 3×3 Matrix, expand along the first row using co-factors. This involves applying the 2×2 rule three times.

Why It Matters

- If $\mathbf{\det(A) \neq 0}$, the matrix has an inverse.
- If $\mathbf{\det(A) = 0}$, it is singular — meaning the system of equations it represents has **no unique solution** or **infinitely many**.

Square Matrix Only!

- Only square matrices have determinants.
- Rectangular matrices (e.g., 2×3) do not have determinants.
- ✓ Valid: 2×2, 3×3, 4×4
- ✗ Invalid: 2×3, 3×4

What is a square matrix:-
One which have same number of rows and columns like 2×2, 3×3
What is a Rectangle matrix:-
One which have different number of rows and columns like 2×3, 3×1, 2×1

How to Calculate it For 2×2 Matrix

Evaluate the determinants of the Matrix A = $\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$

Determinant of the Matrix A would be determined by the following formulae:-

$= a_1 b_2 - a_2 b_1$

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$$

EXAMPLE:-

$$|A| = \begin{vmatrix} 5 & 2 \\ 1 & 7 \end{vmatrix} = 5 \times 7 - 1 \times 2 = 33$$

$$|A| = \begin{vmatrix} 5 & 3 \\ 1 & 6 \end{vmatrix} = 5 \times 6 - 1 \times 3 = 27$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 0 & 7 \end{vmatrix} = 5 \times 7 - 0 \times 2 = 35$$

Let ’s Start the Exercise 4.1

Q1: Evaluate the determinants

$$\begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$$ **Using $a_1 b_2 - a_2 b_1$**

$= 2(-1)-4(-5)$

$=-2+20$

$=18$

Q2: Evaluate the determinant

(i) $\begin{bmatrix} \text{Cos}\theta & -\text{Sin}\theta \\ \text{Sin}\theta & \text{Cos}\theta \end{bmatrix}$

Answer:

$$\begin{bmatrix} \text{Cos}\theta & -\text{Sin}\theta \\ \text{Sin}\theta & \text{Cos}\theta \end{bmatrix}$$

$$=(\text{Cos}\theta)(\text{Cos}\theta)-(-\text{Sin}\theta)(\text{Sin}\theta)$$

$$=\text{Cos}^2\theta+\text{Sin}^2\theta$$

$$=1$$

(ii) $\begin{bmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{bmatrix}$

$$\begin{bmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{bmatrix}$$

$$= (x^2-x+1)(x+1)-(x-1)(x+1)$$

$$= x^3-x^2+x+x^2-x+1-(x^2-1)$$

$$= x^3+1-x^2+1$$

$$= x^3-x^2+2$$

Q3: If $A=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$, then show that $|2A| = 4|A|$

Answer:- The given matrix is $A = \begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$

$\therefore 2A = 2\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix} = \begin{bmatrix}2 & 4\\8 & 4\end{bmatrix}$

$\therefore$ L.H.S. $= |2A| = \begin{vmatrix}2 & 4\\8 & 4\end{vmatrix} = 2 \times 4 - 4 \times 8 \quad = 8 - 32 \quad = -24$

Now, $|A| = \begin{vmatrix}1 & 2\\4 & 2\end{vmatrix} = 1 \times 2 - 2 \times 4 \quad = 2 - 8 \quad = -6$

$\therefore$ R.H.S. $= 4|A| \quad = 4 \times (-6) \quad = -24$

$\therefore$ R.H.S. = L.H.S.

- **|2A|** means first multiplying the matrix with 2 and then finding the determinant.
- **4|A|** means first finding the determinant and then multiplying it with 4

How to Calculate it For a 3 × 3 Matrice.

- Expand along the first row (common method).
- Use minors and cofactors.
- Choose a row/column with more zeros for simplicity.
- You’ll get the same result from expanding any row or column.

If A $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ **is a square matrix of order 3, then**

[Expanding along first row]

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

By removing the row and column in which a_{11} exists.

By removing the row and column in which a_{12} exists.

By removing the row and column in which a_{13} exists.

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

- a_{11} , a_{12}, a_{13} are called the Cofactors and
- The cofactor will have a – sign if the sum of i+j =odd
- a_{11}, i+j = 1+1, the sum is even, 2 - so have a positive sign.
- a_{12} , i+j = 1+2, the sum is odd, 3 - so have a negative sign.

- **That means while expanding along Row 1**
  a_{11} , a_{12}, a_{13} (a_{12} will have negative sign)
- **That means while expanding along Row 2**
  a_{21} , a_{22}, a_{23} (a_{21} , a_{23} will have negative sign)
- **That means while expanding along Row 3**
  a_{31} , a_{32}, a_{33} (a_{32} will have negative sign)

$$\begin{bmatrix} + & - & + \\ - & + & - \\ - & - & + \end{bmatrix}$$

Do we get the same value of Determinant if we expand through any other row or any other column?

- Yes, that's a fundamental property of determinants. We will obtain the same value regardless of which row or column, we use for the expansion.
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- This provides flexibility in calculating determinants, allowing us to choose rows or columns with more zeros to simplify the calculation.

- For example, in Q4 of exercise 4.1 we will expand along first column as it has the maximum zero.

Let's, Find the value of Determinant for a 3×3 Matrix with real values.

Q4: If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer:- The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

It can be observed that in the first column, two entries are zero. Thus , we expand along the first column (C_1) for easier calculation.

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1\ (4-0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \qquad \ldots(i)$$

Now, $3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$\therefore\ |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$

$= 3(36-0)$

$= 3(36)$

$= 108 \qquad \ldots(ii)$

From equations (i) and (ii), we have :

$|3A| = 27|A|$

Hence, the given result is proved.

Q5: Evaluate the determinants

(i) $\begin{bmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Answer:- (i) Let A = $\begin{bmatrix} 3 & -1 & -2 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

It can be observed that in the second row, two entries are zero, thus we expand along the second row for easier calculation

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}$$

= (-15+3)

= -12

(ii) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 3 & 3 & 1 \end{bmatrix}$

By expanding along the first row, we have:

$$|A| = 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} +4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} +5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$= 3(1+6)+4(1+4)+5(3\text{-}2)$
$= 3(7)+4(5)+5(1)$
$=21+20+5= 46$

(iii) $\begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

Let $A=\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

By expanding along the first row, we have:

$$|A| = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} -1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} +2\begin{vmatrix} -1 & 0 \\ -2 & -3 \end{vmatrix}$$

$= 0-1(0-6)+2(-3-0)$

$=-1(-6)+2(-3)$

$=6-6=0$

(iv) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

(iv) Let $A= \begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

By expanding along the first row, we have:

$$|A| = 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$$

= 2(0-5)-0+3(1+4)

= -10+15=5

Q6: If A= $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 4 & 4 & -9 \end{bmatrix}$, find $|A|$.

Answer Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 4 & 4 & -9 \end{bmatrix}$

By expanding along the first row, we have:

$|A| = 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

$=1(-9+12)-1(-18+15)-2(8-5)$

$=1(3)-1(-3)-2(3)$

$= 3+3-6$

$=6-6$

$=0$

Q7: Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

Answer:- (i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow (2 \times 1) - (5 \times 4) = (2x) \times (x) - (6 \times 4)$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow$ (2 x 5) – (3 x 4) = (x x 5) – 3 x 2 x

$\Rightarrow$10-12 = 5 x -6

$\Rightarrow$ -2 = - x

$\Rightarrow$ x = 2

Q8:If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$,
then x is equal to
(A) 6 (B) ± 6 (C) -6 (D) 0

Answer: B

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the correct answer is B.

Useful Properties of Determinants

- $|kA| = k^n \times |A|$ (for n × n matrix)
- $|AB| = |A| \times |B|$
- $|A^t| = |A|$ (Transpose doesn't change the determinant)